Instituut voor Theoretische Fysica
Prof. Dr. G. 't Hooft
Princetonplein 5, Postbus 80.006
3508 TA Utrecht
Telefoon (030) 53 22 84
Telex 40048 fylut nl

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Dear Mr Moroz,

Thank you for your letter and the paper. Indeed, the Green's functions in a (massless) field theory are not merely given by path integrals, but in addition require (two) more limiting procedures

1) Removal of the UV cutoff, giving the UV-renormalon singularities. This makes any non-asymptotically free theory ill-defined*) so that we would not even know how to define a Borel sum unambiguously. Also for asymptotically free theories the renormalon energies, but left of the origin and therefore is more harmless

2) Removal of the IR cutoff, or: infinite volume (thermodynamic) limit. This gives IR renormalons in lethal places. Dealing with them requires knowledge on the vacuum structure of the theory beyond perturbation expension. The problem becomes a circular one that I cannot solve.

I am sending you a reprint (Comm Math Phys 86 (1982) 449 and a preprint of Oct 82 (CMP 88 (1983) Unfortunately I have no (p)reprints available of

*) because of the Landau ghost.

that I consider the best presentation, where I also discuss the Borel transform of these theories: "Planar Diagram Field Theories", in "Progress in Gauge Field Theory", Cargese 1983, Nato ASI series, G. 't Hooft et al ed., Plenum Press, p. 271.

What seems to be missing in your paper is something one could do: find how Borel summability is transported in equations such as the Schwinger-Dyson equations for Green functions one easily finds that branch cuts emerge, and that the renormalization group generates "renormalons" etc etc. Look for instance at recursive equations such as:

$$\Gamma_2(p) = \Gamma_0(p) + g\int dk\ \frac{\Gamma_3(p,k)}{\Gamma_2(p+k)} + \cdots$$

and watch how the singularities in the Borel transform propagate as cuts.

Yours sincerely,

G 't Hooft